# Systems of Linear Equations and Inequalities: Graphing 

## Vocabulary and Key Concepts

Numbers of Solutions of Systems of Linear Equations
different slopes


The lines $\square$ so there is
$\square$ solution.
same slope
different $y$-intercepts


The lines $\square$ so there are
$\square$ solutions.


The lines are $\square$ so there are

solutions.




## Vocabulary:

A system of linear equations is $\qquad$

A solution of a system of linear equations is $\qquad$

Points of Intersection (POI) are the same thing as the solutions of a system.

No solution means $\qquad$
$\qquad$

A system of equations has infinitely many solutions when $\qquad$
$\qquad$

Solve the system graphically. Confirm the solution algebraically.
Ex. 3) $\left\{\begin{array}{l}2 x-6=y \\ 3-x=y\end{array}\right.$



Solve the system graphically. Confirm the solution algebraically.
Ex. 4) $\left\{\begin{array}{l}x-y=6 \\ y=-2 x\end{array}\right.$
Try-It) $\left\{\begin{array}{l}2 x-y=1 \\ 3 x+y=-6\end{array}\right.$



$$
\text { Try-It) }\left\{\begin{array}{l}
2 x-y=5 \\
x-y=1
\end{array}\right.
$$

Try-It) $\left\{\begin{array}{l}-2 x+y=-5 \\ y=2-\frac{1}{3} x\end{array}\right.$



Solve the system graphically. Confirm the solution algebraically.

Ex. 2) $\left\{\begin{array}{l}x=2 \\ y=-6\end{array}\right.$


Try-It) $\left\{\begin{array}{l}y=3 \\ x=-4\end{array}\right.$


Solve the system by graphing in your calculator

1. $y-3 x=7 \quad y+2 x=2$
2. $3 x-y=9 \quad x+2 y=10$
3. $4 x-3 y=-4 \quad-3 x+5 y=-8$

Sometimes, problems involve two linear equations that have to be solved simultaneously. The task is to find one pair $(x, y)$ of values that satisfies both linear equations.
Students in the Hamilton High School science club faced this kind of problem when they tried to raise \$240 to buy a special eyepiece for the high-powered telescope at their school. The school PTA offered to pay club members for an after-school work project that would clean up a nearby park and recreation center building.
Because the outdoor work was harder and dirtier, the deal with the PTA would pay $\$ 16$ for each outdoor worker and $\$ 10$ for each indoor worker. The club had 18 members eager to work on the project. But, most members would prefer the easier indoor work.
$16 x+10 y=240$ and $x+y=18$
a What do the variables $x$ and $y$ represent in these equations?
b What problem condition is represented by each equation?
c What are some combinations of numbers of outdoor and indoor workers that will allow the club to earn just enough money to buy the telescope eyepiece? Will any of those combinations also put each willing club member to work?
d What different strategies could you use to find a pair of values for $x$ and $y$ that satisfy both linear equations simultaneously?

Work on the problems of this lesson will develop your skill in writing, interpreting, and solving systems of linear equations.

## Investigation 1 Solving with graphs and substitution

There are several different methods for solving systems of linear equations. As you work on the problems of this investigation, look for answers to this question:

How can graphs and algebraic substitution be used to solve systems of linear equations?
As you discussed in the Think About This Situation, a system of linear equations expressing the conditions in the science club's situation is:

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16x+10y=240 and x+y=18
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The first equation shows that the amount of money they can earn is a linear function of the variables $x$ and $y$, where $x$ and $y$ represent the number of outdoor and indoor workers, respectively. The second equation shows that the number of club members who will work is also a linear function of those variables.

The diagram below shows graphs (in the first quadrant) of solutions to the equations $16 x+10 y=240$ and $x+y=18$.

a. Match the graphs to the linear equations they represent. Explain how you know that your answers are correct.
b. Use the graphs to estimate a solution $(x, y)$ for the system of equations-values for $x$ and $y$ that satisfy both equations. what the solution tells about the science club's fund-raising situation.
c. Since graphs give only estimates for solutions of equations, it is important to check the estimates. Show how your graph-based estimate can be checked to see if it is an exact solution to the system.
2. When the date for the work project was set, it turned out that only 13 science club members could participate. The club president talked again with the PTA president and got a new pay deal-\$20 per outdoor worker and $\$ 15$ per indoor worker.
a. Write a system of linear equations in which one equation expresses the new conditions about payment and the other shows the new number of workers.
b. Estimate the solution for this system of equations by using graphs of the two equations. Then check your estimate.


## Systems with No solutions

$$
\text { 1.): }\left\{\begin{array}{l}
y=3 x+2 \\
y=3 x-2
\end{array}\right.
$$



## Systems with Infinitely Many solutions

2. $\left\{\begin{array}{l}y=-\frac{3}{4} x+3 \\ y=-\frac{3}{4} x+3\end{array}\right.$



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| Main Ideas | Details |
| :---: | :---: |
|  | Fitness Competition <br> - Carly is training for an upcoming fitness competition and is trying to find a breakfast combination that meets her nutritional requirements of $\mathbf{1 5 0 0}$ calories and $\mathbf{5 0}$ grams of protein. One serving of her cereal of choice has $\mathbf{2 5 0}$ calories and $\mathbf{5}$ grams of protein. Her favorite brand of peanut butter contains $\mathbf{2 5 0}$ calories and $\mathbf{1 0}$ grams of protein per serving. <br> - Write a system of equations to find the number of servings for each type of food that would meet both of her nutrition goals. Use your calculator to complete this task. |
|  | Airplane Tickets <br> - Laura and Andy are trying to earn money to buy airplane tickets to visit their favorite aunt, Annie. Laura's ticket is going to cost her $\mathbf{\$ 2 8 0}$ to visit their favorite aunt, Annie. Andy's ticket is going to cost him \$230 money, they have both decided to mow lawns and babysit. Laura charges $\$ 7$ per hour for babysitting while Andy charges $\$ \mathbf{5}$ per hour. To mow a lawn, Laura charges $\$ 14$ per lawn while Andy charges $\$ 16$ per lawn. <br> - Write a system of equations to find the number of hours each needs to babysit and to find how many lawns they each need to mow. Use your calculator to complete this task. |

1. Graph $\begin{aligned} & y \geq 2 \\ & x<-3\end{aligned}$.
\#2. Graph $y \geq \frac{1}{2} x$.

\#5. Graph $\begin{aligned} & 3 x-4 y>2 \\ & 3 x-y \geq 2\end{aligned}$



| Main Ideas | Details |  |
| :---: | :---: | :---: |
| Write a system of Linear Inequalities that describes the graph |    <br> Match the system of linear inequalities with its graph. <br> A. $x+y \leq 4$ $x+y \geq-2$ <br> 1. <br> B. $\begin{aligned} x+2 y & \leq 4 \\ -2 x+y & \geq-2 \end{aligned}$ <br> 2. | c. $x+y \geq-2$ $-2 x+y \geq-2$ <br> 3. |

# Systems of Linear Equations and Inequalities: Algebraically Substitution and Elimination 



Solving Systems Algebraically - set them equal \& substitution
Graph the following system of equations, find the solution.

1. $\left\{\begin{array}{l}y=2 x+6 \\ y=x+3\end{array}\right.$
2. $\left\{\begin{array}{l}y=2 x+6 \\ y=x+3\end{array}\right.$


## Try It! - Solve Algebraically

a. $\quad\left\{\begin{array}{l}y=3 x-30 \\ y=-x+14\end{array}\right.$
b. $\quad\left\{\begin{array}{l}x=-4 y+1 \\ x=y-4\end{array}\right.$


| Main Ideas | Details |
| :--- | :--- |
|  | Trey's online music club charges a monthly rate of $\$ 20$ plus $\$ 0.80$ per song <br> download. Deb's online music club charges a monthly rate of $\$ 21$ plus $\$ 0.60$ per <br> song download. For what number of songs will the monthly charge be the same <br> for both clubs? How much will it cost? |
|  | One car model costs $\$ 12,000$ and costs an average of $\$ 0.10$ per mile to maintain. <br> Another car model costs $\$ 14,000$ and costs an average of $\$ 0.08$ per mile to <br> maintain. If one of each model is driven the same number of miles, after how <br> many miles would the total cost be the same? |
|  |  |




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A system of linear equations expressing the conditions in the science club's situation is:

## $16 x+10 y=240$ and $x+y=18$

The first equation shows that the amount of money they can earn is a linear function of the variables $x$ and $y$, where $x$ and $y$ represent the number of outdoor and indoor workers, respectively. The second equation shows that the number of club members who will work is also a linear function of those variables.
a. Use the substitution method to find the solution $(x, y)$ for the system of equations-values for $x$ and $y$ that satisfy both equations. What the solution tells about the science club's fund-raising situation.
2. When the date for the work project was set, it turned out that only 13 science club members could participate. The club president talked again with the PTA president and got a new pay deal-\$20 per outdoor worker and $\$ 15$ per indoor worker.
a. Write a system of linear equations in which one equation expresses the new conditions about payment and the other shows the new number of workers.
b. Use the substitution method to find the solution $(x, y)$ for the system of equations-values for $x$ and $y$ that satisfy both equations. What the solution tells about the science club's fund-raising situation.

| Special Solutions Substitution Method | Details |  |  |
| :---: | :---: | :---: | :---: |
|  | 1. $\left\{\begin{array}{l}y=2 x+3 \\ y=2 x+2\end{array}\right.$ |  |  |



| Main Ideas | Details |
| :---: | :---: |
|  | Solve each of the following systems using the substitution method.(Toolkit) <br> a. $x-y=4 \quad 2 x-2 y=8$ <br> b. $\quad 3 x-6 y=-46.5 \quad-x+2 y=15.5$ |


| Main Ideas | Details |
| :---: | :---: |
|  | Art fundraiser <br> - The Kesling Middle School booster club is planning a community event to raise money for the school's art department. <br> - Based on previous fund-raising events, they estimate that the event will be a sellout-filling all 400 seats. Plans are to charge adults $\$ 10$ and children $\$ 5$.The club wants to earn $\$ 3000$. <br> Write a system of linear equations and solve using the substitution method |

Solving by Elimination/Addition Method Examples:

1. $\left\{\begin{array}{l}x+y=3 \\ x-y=-9\end{array}\right.$
2. $\left\{\begin{aligned} 2 x-4 y & =10 \\ -2 x+6 y & =-4\end{aligned}\right.$

Try It!
a. $\left\{\begin{array}{r}2 x+y=3 \\ -2 x+y=1\end{array}\right.$
b. $\left\{\begin{array}{l}x+y=30 \\ x-y=6\end{array}\right.$

## Examples:

3. $\left\{\begin{aligned} 6 x-7 y & =-4 \\ -4 x-7 y & =26\end{aligned}\right.$
4. $\left\{\begin{array}{l}x+3 y=9 \\ x-2 y=-6\end{array}\right.$

Try It!
a. $\left\{\begin{array}{l}5 x+7 y=77 \\ 5 x+3 y=53\end{array}\right.$
b. $\left\{\begin{array}{l}9 x-3 y=24 \\ 7 x-3 y=20\end{array}\right.$

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Now let's investigate some other systems that involve other uses of the elimination method.
5. $\left\{\begin{aligned} 2 x+5 y & =-1 \\ x+2 y & =0\end{aligned}\right.$
6. $\left\{\begin{array}{l}6 x+3 y=0 \\ -3 x+3 y=9\end{array}\right.$

Try It!
a. $\quad\left\{\begin{array}{l}8 x-9 y=19 \\ 4 x+y=-7\end{array}\right.$
b. $\left\{\begin{array}{l}4 x-y=6 \\ 3 x+2 y=21\end{array}\right.$

## Examples:

1. $\left\{\begin{array}{l}3 x+5 y=10 \\ 5 x+7 y=10\end{array}\right.$
2. $\left\{\begin{array}{l}15 x+3 y=9 \\ 10 x+7 y=-4\end{array}\right.$

Try It!
a. $\quad\left\{\begin{array}{l}2 x-3 y=-11 \\ 3 x+2 y=29\end{array}\right.$
b. $\left\{\begin{array}{l}5 x+7 y=-1 \\ 4 x-2 y=22\end{array}\right.$

| Main Ideas | Details |
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A system of linear equations expressing the conditions in the science club's situation is:

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b. Use the elimination method to find the solution $(x, y)$ for the system of equations-values for $x$ and $y$ that satisfy both equations. What the solution tells about the science club's fund-raising situation.

| Main Ideas | Details |
| :---: | :---: |
|  | Fitness Competition <br> - Carly is training for an upcoming fitness competition and is trying to find a breakfast combination that meets her nutritional requirements of $\mathbf{1 5 0 0}$ calories and $\mathbf{5 0}$ grams of protein. One serving of her cereal of choice has $\mathbf{2 5 0}$ calories and $\mathbf{5}$ grams of protein. Her favorite brand of peanut butter contains $\mathbf{2 5 0}$ calories and $\mathbf{1 0}$ grams of protein per serving. <br> - Write a system of equations to find the number of servings for each type of food that would meet both of her nutrition goals. |

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For some people, like athletes and astronauts selection of a good diet is a carefully planned scientific process. In the case of astronauts, proper nutrition is provided in limited forms. For example, drinks might come in disposable boxes and solid food in energy bars. Suppose that, in planning daily diets for a space shuttle team, nutritionists work toward these goals.

- Drinks each provide 30 grams of carbohydrate, energy bars each provide 40 grams of carbohydrate, and the optimal diet should contain 600 grams of carbohydrate per day.
- Drinks each provide 15 grams of protein, energy bars each provide 20 grams of protein, and the optimal diet should contain200 grams of protein.

The problem is to find a number of drinks and a number of energy bars that will provide just the right nutrition for each astronaut. If we use $x$ to represent the number of drinks and $y$ for the number of energy bars, the goals in diet planning can be expressed as a system of linear equations:
$30 x+40 y=600 \quad 15 x+20 y=200$

Solve the system using the elimination method.

Suppose that the condition on protein in Problem 2 was revised to require 300 grams per day.
a. Write the new system of equations expressing the conditions relating number of drink boxes and number of food bars to the required grams of carbohydrate and protein in the diet.
b. Solve the system using the elimination method.


## Linear Programming

1. A company makes backpacks and briefcases. Daily output cannot exceed a total of 50 backpacks and briefcases. A maximum of 30 backpacks can be made in one day. The maximum daily output of briefcases is 20 .
a) Define variables $x$ and $y$ for this problem.

Let $x$ represent $\qquad$
Let $y$ represent $\qquad$
b) State the constraints given in this problem. Write an inequality for each constraint.
c) Find the $x$ and $y$-intercepts for the constraints above.
d) Draw a graph that shows the possible numbers of bags that can be made in 1 day. Label the axis. Use a scale of 5 on both axes.


One possible solution is ( , )
Show that this works in all 3 inequalities
2. Alyssa plays soccer and baseball. She burns 600 calories/h playing soccer and 60
calories/h playing baseball. Each week she is willing to spend at most 30 h exercising and wishes to burn at least 5000 calories.
a) Define variables $x$ and $y$ for this problem.

Let $x$ represent $\qquad$ Let $y$ represent $\qquad$
b) Write a system of inequalities to represent the constraints in this problem.
c) Find the $x$ and $y$ - intercepts for the constraints above.
d) Draw a graph to show the time Alyssa could spend on each activity in one week.

Label the axis. Use a scale of 2 on the $\mathbf{x}$ - axis and 5 on the $y$-axis


One possible solution is ( , )
Show that this works in both inequalities

| Main Ideas | Details |
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## System of Inequalities - Walk through

1. You can work at most 30 hours next week. You need to earn at least $\$ 150$ to cover you weekly expenses. Your dog- walking job pays $\$ 10$ per hour and your job as a car wash attendant pays $\$ 5$ per hour.
a) Define the variables.
$\qquad$ $=\mathrm{x}$
$\qquad$ $=\mathrm{y}$
b) Write your constraints and solve as needed.

c) Graph your constraints. Identify your solution.
2. Marsha is buying plants and soil for her garden. The soil cost $\$ 5$ per bag, and the plants cost $\$ 8$ each. She wants to buy at least 8 plants and can spend no more than $\$ 150$.
a) Define the variables.
$\qquad$

b) Write your constraints and solve as needed.
c) Graph your constraints. Identify your solution.
3. The area of a parking lot is 1000 square meters. A truck requires 10 square meters. A bus requires 25 square meters. The attendant can handle at most 50 vehicles.
b) Define the variables.
$\qquad$
$\qquad$ $=\mathrm{y}$
c) Write your constraints and solve as needed.

d) Graph your constraints. Identify your solution.
4. A theater wants to take in at least $\$ 1800$ for a certain matinee. Children's tickets cost $\$ 6$ each and adult tickets cost $\$ 9$ each. The theater can seat up to 400 people.
a) Define the variables.

$\qquad$
= x
$\qquad$ = y
b) Write your constraints and solve as needed.
c) Graph your constraints. Identify your solution.
5. Oaken Treasures make two different kinds of chairs, rockers, and swivels. Work on machines A and B is required to make both kinds. Machine A can be run no more than 30 hours a day. Machine B is limited to 20 hours a day. The following chart shows the amount to time on each machine that is required to make one chair.

| Chair | Operation A | Operation B |
| :---: | :---: | :--- |
| Rocker | 1 h | 2 h |
| Swivel | 3 h | 2 h |

a) Define the variables.

$\qquad$
b) Write your constraints and solve as needed.
c) Graph your constraints. Identify your solution.
6. A delivery service has a fleet of trucks. Each truck can accommodate a maximum weight of 4000 lb and 450 cubic ft of merchandise. A small refrigerator weights 20 lb and occupies 5 cubic ft of space. A 42-in television set weighs 40 lb and occupies 9 cubic ft of space.
a) Define the variables.

$\qquad$
$\qquad$
b) Write your constraints and solve as needed.
c) Graph your constraints. Identify your solution.

## System of Inequalities - Word Problems

An office manager is purchasing file cabinets (in cubic feet). The office has 100 square feet of floor space for the cabinets and $\$ 1000$ in the budget to purchase them. Cabinet A requires 5 square feet of floor space, and costs $\$ 100$ dollars. Cabinet B requires 10 square feet of floor space, and costs $\$ 150$. Define the variables.
$\qquad$
d) Write your constraints and solve as needed.

e) Graph your constraints. Identify your solution.
2. Superbats, Inc., manufactures two different quality wood baseball bats, the Wallbanger and the

Dingbat. The Wallbanger takes 6 hours to trim and turn on a lathe and 3 hours to finish it. The Dingbat takes 8 hours to trim and
 turn on a lathe and 2 hours to finish. The total time per day available for trimming and lathing is 96 hours and for finishing is 60 hours.

Define the variables.
$\qquad$
= x
b) Write your constraints and solve as needed.
c) Graph your constraints. Identify your solution.
3. A ski company makes two types of skis and has a fabrication and a finishing department. A pair of downhill skis requires 8 hours to fabricate and 2 hour to finish. A pair of cross-country skis requires 6 hours to fabricate and 1 hour to finish. The fabricating department has 96 hours of labor available per day. The finishing department has 40 hours of labor available per day.
e) Define the variables.
$\qquad$
$\qquad$
f) Write your constraints and solve as needed.

g) Graph your constraints. Identify your solution.
4. Juan makes two types of wood clocks to sell at local stores. It takes him 4 hours to assemble a pine clock, which requires 2 oz of varnish. It takes 5 hours to assemble an oak clock, which takes 6 oz . of varnish. Juan has 24 oz . of varnish in stock, and can work 40 hours.

a) Define the variables.
$\qquad$
$\qquad$
b) Write your constraints and solve as needed.
b) Graph your constraints. Identify your solution.

| Main rdeas | Details |
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|  |  |
|  | Linear |
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| Programming with |  |
| Optimization |  |



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| The Cruiser Bicycle Company makes two styles of bicycles: The <br> Traveler, which sells for $\$ 300$ and the Tourister, which sells for <br> \$600. Each bicycle has the same frame and tires, but the <br> assembly and painting time required for the Traveler is only <br> 1 hour while it is 3 hours for the Tourister. There are 300 frames <br> and 360 hours of labor available for production. How many <br> bicycles of each model should be produced to maximize <br> revenue? <br> Define the variables. <br> x= <br> Write your constraints and solve as needed. |
| :--- | :--- |

